PROBLEM SET 05 - Recurrence relations 1.

1 Finding recurrence relation

Exercise 1.1. Find a recurrence relation for the number of permutations of $[n]$.

Exercise 1.2. A species of bacteria reproduces in the following way. Each newborn bacteria at the end of the first day creates its two copies and at the end of every other day it gives rise to 3 newborn bacteria. Give a recurrence formula for a_n–the number of bacteria after n days if at the beginning there are 5 newborn bacteria.

Exercise 1.3. Find a recurrence relation for the number of ways to completely cover a
    a) $2 \times n$ checkerboard
    b) (*) $3 \times 2n$ checkerboard
with 1×2 (indistinguishable) dominoes.

Exercise 1.4. There are $2^{n-1} - 1$ unordered divisions of the set $[n]$ into two non–empty sets. Find a recurrence formula for a_n – the number of unordered divisions of the set $[n]$ into three non–empty sets.

Exercise 1.5. Let a_n be the number of subsets of the set $[n] = \{1, \ldots, n\}$, which do not have
    a) two consecutive numbers.
    b) pairs of the type $k, k + 2$.
Find a recurrence formula for a_n.

Exercise 1.6. A vending machine dispensing books of stamps accepts only one-dollar coins, \$1 bills, and \$5 bills. Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.

Exercise 1.7. Messages are transmitted over a communication channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds. Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.

Exercise 1.8. A country uses as currency coins with values of 1 peso, 2 pesos, 5 pesos, and 10 pesos and bills with values of 5 pesos, 10 pesos, 20 pesos, 50 pesos, and 100 pesos. Find a recurrence relation for the number of ways to pay a bill of n pesos if the order in which the coins and bills are paid matters.

Exercise 1.9. How many ways are there to put flags on a n meters high mast if we have any number of

    a) indistinguishable red flags of length 2, indistinguishable green flags of length 1, and indistinguishable blue flags of length 1;

    b) indistinguishable red flags of length 1 and indistinguishable green flags of length 1 and we have no two green subsequent flags.

Exercise 1.10. Each year an employee gets a rise equal to 20% of the salary of the prior year minus 11% of the salary paid two years ago. At the begining the employee gets one thousand dollars. Find a recurrence relation for a_n – salary of the employee after n years.

Exercise 1.11. In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces?