PROBLEM SET 04 - The principle of inclusion-exclusion

1 Two or three sets

Exercise 1.1. How many bit strings of length eight either start with a 1 bit or end with two bits 0?

Exercise 1.2. An IT company received 350 applications from university graduates. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

Exercise 1.3. How many permutations of the 26 letters of the English alphabet do not contain any of the strings FISH, RAT or BIRD?

Exercise 1.4. In a discrete mathematics class every student is a major in computer science or mathematics, or both. The number of students having computer science as a major (possibly along with mathematics) is 25; the number of students having mathematics as a major (possibly along with computer science) is 13; and the number of students majoring in both computer science and mathematics is 8. How many students are in this class?

Exercise 1.5. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 1000 in A_2, and 10 000 in A_3 if

a) $A_1 \subseteq A_2$ and $A_2 \subseteq A_3$.

b) the sets are pairwise disjoint.

c) there are two elements common to each pair of sets and exactly one element belonging to all three sets.

Exercise 1.6. Find the number of positive integers not exceeding 10000 that are either odd or the square of an integer.

Exercise 1.7. How many solutions to the equation

$$x_1 + x_2 + x_3 = 11$$

are there, where x_1, x_2, and x_3 are nonnegative integers with $x_1 \leqslant 6$, $x_2 \leqslant 6$, and $x_3 \leqslant 6$?

Exercise 1.8. How many ways are there to distribute six different toys to three different children such that each child gets at least one toy?

2 General case

Exercise 2.1. How many ways are there to write a word consisting of 26 pairs of identical letters (A A B B C C . . . Z Z)

a) with at least one pair of subsequent identical letters?

b) with no subsequence of identical letters?

Exercise 2.2. In how many ways can the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 be arranged so that no even digit is in its original position?

Exercise 2.3. How many elements are in the union of five sets if the sets contain 10,000 elements each, each pair of sets has 1000 common elements, each triple of sets has 100 common elements, every four of the sets have 10 common elements, and there is 1 element in all five sets?

Exercise 2.4. How many permutations of a deck of 52 cards are there if

a) there is at least one pair of subsequent Queens;

b) there are no two subsequent **black** cards of the same value (values are: $2, 3, \ldots, 10, Kn, Q, Ki, A$).

Exercise 2.5. How many ways are there to pick 20 cards from a deck of 52 if each of the values: 2, 3, 4, . . . , 9 is chosen (at least once).

Exercise 2.6. Cards from a deck of 52 cards are put into 13 undistinguishable boxes and each box contains exactly 4 cards. In how many ways one can do it (order of the cards in a box is not relevant) if none of the boxes contains all four cards of the same value (values are: 2, . . . , 10, Kn, Q, Ki, A).

Exercise 2.7. A group of n students is assigned seats for each of the two classes in the same classroom. In how many ways can these seats be assigned if no student is assigned the same seat twice?

Exercise 2.8. How many ways are there to assign 20 different jobs to 9 different employees if

a) at least one employee is without a job?

b) every employee is assigned at least one job?