PROBLEM SET 03 - Selection schemes

1 Selection schemes

NAME SELECTION SCHEMES USED IN THE SOLUTIONS

Exercise 1.1. How many ways are there to distribute 10 indistinguishable balls into 22 distinguishable bins if
a) each bin may contain at most one ball?
b) each bin may contain unlimited number of balls?
What is the answer, when balls are distinguishable?

Exercise 1.2. How many bit strings of length 10 contain
a) exactly four 1s?
b) at most four 1s?
c) at least four 1s?
d) an equal number of 0s and 1s?

Exercise 1.3. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

Exercise 1.4. How many subsets with more than two elements does a set with 100 elements have?

Exercise 1.5. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
a) are there in total?
b) contain exactly two heads?
c) contain at most three tails?
d) contain the same number of heads and tails?

Exercise 1.6. How many permutations of the letters ABCDEFG contain
a) the string BCD?
b) the strings BA and GF?

Exercise 1.7. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

Exercise 1.8. Thirteen people on a softball team show up for a game.
a) How many ways are there to choose 10 players to take the field?
b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Exercise 1.9. The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
a) exactly one vowel?
b) exactly two vowels?
c) at least one vowel?
d) at least two vowels?

Exercise 1.10. How many bit strings contain exactly eight 0s and ten 1s if every 0 must be immediately followed by a 1?

Exercise 1.11. A circular r–permutation of n people ($r \leqslant n$) is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table. What is the number of circular r–permutations of n people?

Exercise 1.12. Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

Exercise 1.13. How many different ways are there to choose 12 donuts from the 21 varieties at a donut shop?

Exercise 1.14. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels (bagels of a given type are indistinguishable). How many ways are there to choose
a) 12 bagels?
b) 12 bagels with at least one of each kind?
c) 12 bagels with at least three egg bagels and no more than two salty bagels?

Exercise 1.15. How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

Exercise 1.16. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, where x_i , $i = 1, 2, 3, 4, 5$, is a nonnegative integer such that
a) $x_1 \geqslant 1$?
b) $x_i \geqslant 2$ for $i = 1, 2, 3, 4, 5$?
c) $0 \leqslant x_1 \leqslant 10$?

Exercise 1.17. How many strings of ten ternary digits (0, 1, or 2) are there that contain exactly two 0s, three 1s, and five 2s?

Exercise 1.18. How many positive integers less than 1 000 000 have the sum of their digits equal to 9?

Exercise 1.19. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

Exercise 1.20. A student has three mangos, two papayas, and two kiwi fruits. If the student eats one piece of fruit each day, and only the type of fruit matters, in how many different ways can these fruits be consumed?

Exercise 1.21. A professor packs her collection of 40 issues of a Mathematics journal in four boxes with 10 issues per box. How many ways can she distribute the journals if
a) each box is numbered, so that they are distinguishable?
b) the boxes are identical, so that they cannot be distinguished?

Exercise 1.22. How many ways are there to deal hands of seven cards to each of five players from a standard deck of 52 cards?

Exercise 1.23. In how many ways can n books be placed on k distinguishable shelves
a) if the books are indistinguishable copies of the same title?
b) if no two books are the same, and the positions of the books on the shelves do not matter?
c) (*) if no two books are the same, and the positions of the books on the shelves matter?

Exercise 1.24. A shelf holds twelve books in a row. How many ways are there to choose five books so that no two adjacent books are chosen? [Hint: Represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of five bars and seven stars so that no two bars are adjacent.]