PROBLEM SET 02 - Basic counting rules

1 Basic rules

Exercise 1.1. How many functions are there from a set with m elements to a set with n elements?

Exercise 1.2. How many one-to-one functions are there from a set with m elements to one with n elements ($m \leqslant n$)?

Exercise 1.3. What is the number of different subsets of a finite set S?

Exercise 1.4. Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Exercise 1.5. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

Exercise 1.6. In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?
b) both the bride and groom must be in the picture?
c) exactly one of the bride and the groom is in the picture?

Exercise 1.7. How many bit strings of length seven either begin with two 0s or begin with three 1s?

Exercise 1.8. Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. Moreover three different airlines fly from New York to San Francisco directly. How many different configurations of airlines can you choose on which to book a trip from New York to San Francisco and back? What if you do not want to repeat any connection, when you go back?

Exercise 1.9. A multiple–choice test contains 10 questions. There are four possible answers for each question.

a) In how many ways can a student answer the questions on the test if the student answers every question?
b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

Exercise 1.10. How many ways are there to put fruits in a basket, so that the basket is not empty, if we have n indistinguishable apples and m indistinguishable oranges.

2 Bijections

Exercise 2.1. a. A– the set of all bit strings of length n (n–digit binary sequences);
B – the family of all subsets of the set $[n] = \{1, \ldots, n\}$.

b. A– the set of all arrangement of n distinguishable balls in k numbered boxes;
B– the set of sequences of length n with elements from the set $[k] = \{1, \ldots, k\}$;

c. A– the set of arrangements of n identical balls in k ($n \leqslant k$) numbered boxes, such that each box contains at most one ball;
B – the family of k–element subsets of the set $[n] = \{1, \ldots, n\}$

d. A – the set of n–digit binary sequences with exactly k ones ($k \leqslant n$);
B – the family of k–element subsets of the set $[n] = \{1, \ldots, n\}$ ($k \leqslant n$);

e. A – the set of all solutions of the equation

$$x_1 + x_2 + \ldots + x_{2k} = 0, \quad x_i \in \{-1, 1\};$$

B – the set of all shortest paths between opposite corners of the lattice with side of length k;

f. A – the set of all solutions of the equation

$$x_1 + x_2 + \ldots + x_{2k} = 4, \quad x_i \in \{-1, 1\}; \quad k \geqslant 2$$

B – the family of $(k+2)$–element subsets of the set $[2k] = \{1, 2, \ldots, 2k\}$, $k \geqslant 2$.

g. A – the set of all integer solutions of the equation

$$x_1 + x_2 + \ldots + x_k = n, \quad x_i \geqslant 2$$

B – the set of all integer solutions of the equation

$$y_1 + y_2 + \ldots + y_k = n - k, \quad y_i \geqslant 1$$

h. A – the set of all integer solutions of the equation

$$x_1 + x_2 + \ldots + x_k = n, \quad x_i \geqslant 1$$

B – the set of $(n+k-1)$–digit binary string with n zeros and $k-1$ ones, starting and ended by zero with no two consecutive ones.

i. A – the set of all integer solutions of the equation

$$x_1 + x_2 + \ldots + x_k = n, \quad x_i \geqslant 1$$

B – the family of $(k-1)$–element subsets of the $(n-1)$–element set.

j. A – the set of all 6–letter words, which may be created using letters from the word TAMTAM
B – the family of ordered divisions of the set $\{1, 2, 3, 4, 5, 6\}$ into two–element subsets.